Generalized Procrustes Analysis Example with Annotation

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Generalized Procrustes Analysis (GPA) is particularly useful for analyzing repertory grid data collected from numerous individuals or from the same individual on different occasions. In this brief paper the common steps in GPA will be demonstrated using example grids taken from a recent encyclopedia entry by Grice (2007). Consider the following ratings of six employees by four managers:

Manager 1

	John	Bob	Amy	Jan	Fred	Jill
Extraverted	1	1	5	4	5	3
Sharing	5	5	2	1	2	3
Motivated	2	2	4	3	4	1
Funny	1	2	2	2	4	1
Loud	1	2	2	1	4	1

Manger 2

	John	Bob	Amy	Jan	Fred	Jill
Blunt	2	3	1	3	4	1
Patient	5	1	4	2	2	3
Creative	5	2	3	4	1	3
Outgoing	2	4	1	4	3	3

Manager 3

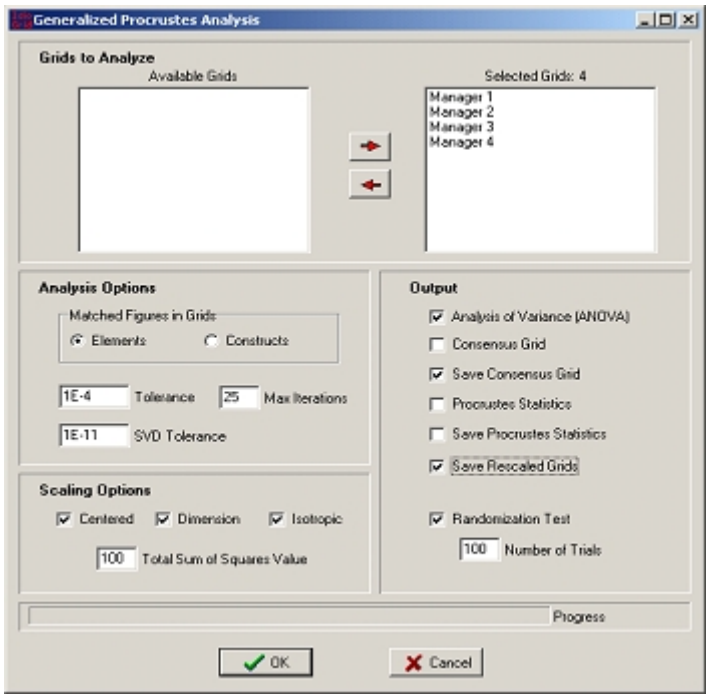
	John	Bob	Amy	Jan	Fred	Jill
Outgoing	1	4	2	4	5	3
Carefree	1	5	3	4	4	2
Generous	4	2	5	2	1	5
Trusting	4	2	4	3	1	4
Organized	5	3	3	2	3	3
Athletic	2	1	1	2	5	1

Manager 4

	John	Bob	Amy	Jan	Fred	Jill
Easy Going	2	3	1	3	4	3
Outgoing	1	3	1	5	5	1
Nurturing	3	3	5	5	1	5
Calm	5	2	4	1	2	5
Intelligent	5	3	5	2	3	3

The managers completed these repertory grids by describing, in their own words (that is, using their own personal constructs), the six employees. Scales (1 to 5 points) were constructed from their descriptions, and each manager then rated each employee on the scales. As can be seen, the managers differed in the number of personal constructs they used to describe the employees. With GPA, such differences are permissible as long as the grids are matched on the elements (that is, the people being rated). More generally, GPA requires either the constructs or the elements to be matched across the grids. Both constructs and elements can be matched as well.

The four grids are included with the latest version (2.4) of the Idiogrid software (‘*Manager1.grd, Manager2.grd*, etc., files), and they can be loaded and analyzed. Once the grids are loaded, the user selects *Analyses –> Generalized Procrustes Analysis* from the Main Menu in Idiogrid. The GPA options window, with the appropriate options selected will appear as follows:



As can be seen, the four grids are selected, and the ‘Matched Figures in Grids’ option is set to ‘Elements’ for the current grids. The default tolerance and iteration values are left unchanged, and all of the scaling options are selected. The ‘Centered’ and ‘Isotropic’ options are commonly selected and set as the defaults in Idiogrid, but the ‘Dimension’ scaling option is selected here as well since the grids have different numbers of constructs (that is, the grids differ in their dimensionality). An ANOVA of the results has also been requested along with a randomization test. The consensus grid will also be saved as a new grid for further analysis, and each manager’s rescaled and rotated grid will be saved for further analysis as well. Selecting ‘OK’ to run the analysis, generates the following output:

Generalized Procrustes Analysis

Grids Analyzed (4)
Manager 1, Manager 2, Manager 3, Manager 4

Centered Scaling Factors

Centering Means

	Manager 1	Manager 2	Manager 3	Manager 4
Con_1	3.17	2.33	3.33	2.67
Con_2	3.00	2.83	3.17	2.67
Con_3	2.67	3.00	3.00	3.67
Con_4	2.00	2.83	3.00	3.17
Con_5	1.83	0.00	3.00	3.50
Con_6	0.00	0.00	2.50	0.00

Note. Prior to analysis, the grid values were centered about these mean values.

Dimensional Scaling Factors
Manager 1 : 2.24
Manager 2 : 2.00
Manager 3 : 2.45
Manager 4 : 2.24

Note. Prior to analysis, the values in each grid were divided by their respective dimensional scaling values.

Lambda Scaling Factor
SS Value : 100
Lambda : 1.67

Note. Prior to anlaysis, the values in each grid were multiplied by Lambda.

Isotropic Scaling Factors
Manager 1 : 0.75
Manager 2 : 1.01
Manager 3 : 1.48
Manager 4 : 0.94

Note. After the initial Procrustes rotations, the values in each grid were multiplied by their respective isotropic scaling values.

Iteration History
1 : -69.87332404583066
2 : -71.14153878390364
3 : -71.15291188773783
4 : -71.15313657697925
5 : -71.15313941934808

[Manager 1 rescaled grid created (see Grid Data window)]
[Manager 2 rescaled grid created (see Grid Data window)]
[Manager 3 rescaled grid created (see Grid Data window)]
[Manager 4 rescaled grid created (see Grid Data window)]

ANOVA Source Table for Matched Figures

Figures	*Consensus*	*Residual*	*Total*
John	16.71	2.33	19.04
Bob	5.47	6.28	11.76
Amy	9.74	5.87	15.61
Jan	11.01	3.55	14.56
Fred	24.10	1.41	25.51
Jill	11.54	1.99	13.52
Total SS:	78.57	21.43	100.00

ANOVA Source Table for Grids

Grids	*Residual*	*Total*
Manager 1	9.87	16.09
Manager 2	5.42	24.61
Manager 3	2.64	29.94
Manager 4	3.51	29.36
Total SS:	21.43	100.00
Consensus Proportion:	0.79	

Specific ANOVA Residuals

	Manager 1	Manager 2	Manager 3	Manager 4
John	0.77	0.75	0.18	0.63
Bob	3.65	1.94	0.62	0.08
Amy	3.39	0.86	0.54	1.07
Jan	0.95	1.31	0.13	1.17
Fred	0.72	0.09	0.21	0.39
Jill	0.38	0.47	0.97	0.17

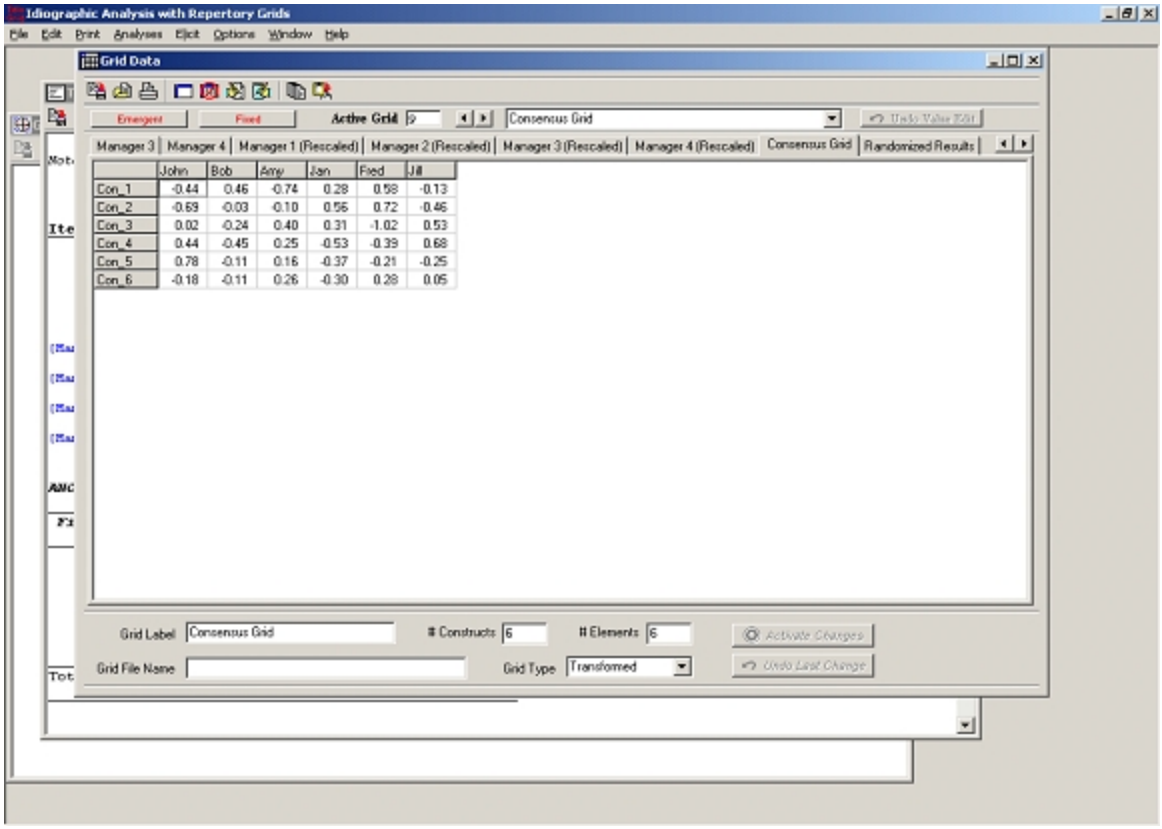
{Consensus Grid created (see Grid Data window)}

Randomization Results

Observed Consensus Proportion	0.79
Number of Random Proportions	100.00
Minimum Random Proportion	0.65
Maximum Random Proportion	0.83
Values > Observed Proportion	5.00
Approximate p-value	0.05

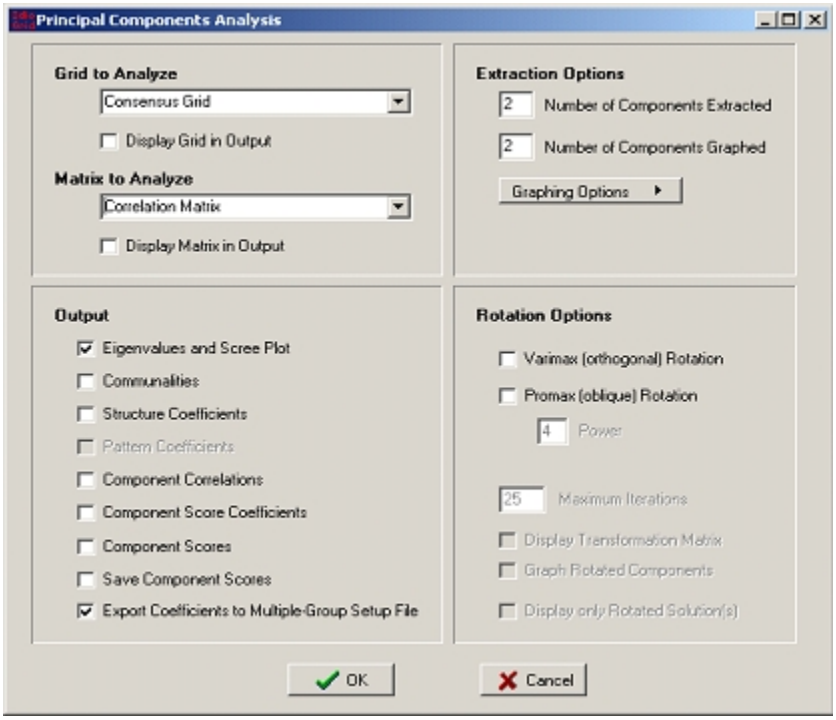
{Randomization Results Grid created (see Grid Data window)}

The consensus grid, as well as the specific results from the Randomization test, are saved as new grids in the grid data window:

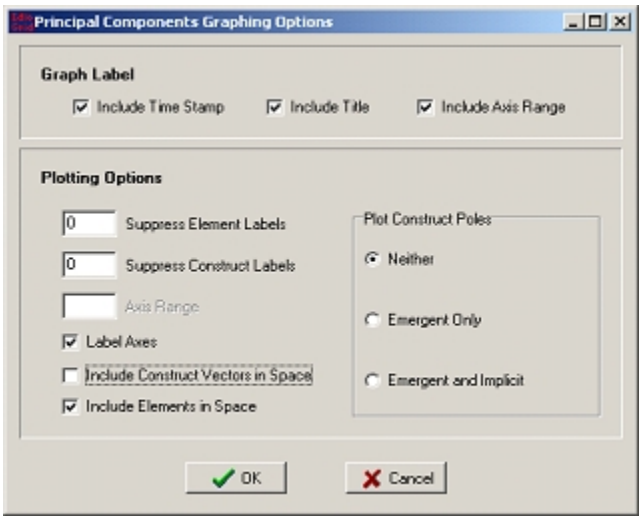


The rows of the consensus grid are labeled ‘Con_1', ‘Con_2', etc. It is important to realize that since the constructs were not matched in the grids, these represent arbitrarily labeled dimensions along which the elements have been ordered in the analysis. ‘Con_1', for instance, is therefore not a construct from a particular manager’s grid. It is simply an abstract dimension resulting from the analysis. As will be shown below, the personal constructs of the managers can be correlated with these dimensions, and the elements (that is, the rated employees) can be mapped into spaces formed from these dimensions as well.

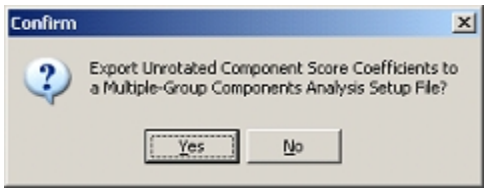
A common strategy is to conduct a Principal Components Analysis on the saved consensus grid. In Idiogrid, the user selects *Analyses –> Principal Components Analysis*. The following options window, with a number of desirable options selected, will open:



As can be seen, consensus grid is selected as the 'Grid to Analyze.' The 'Eigenvalues and Scree Plot' option is chosen, as well as the 'Export Coefficients to Multiple-Group Setup File' option. The correlations of the consensus grid will be analyzed, and the first two principal components will be extracted and plotted. Prior to running the analysis, the 'Graphing Options' button should be selected, and the following options chosen:



As can be seen, the ‘Plot Construct Poles’ option is set to ‘Neither’, and the ‘Include Construct Vectors in Space’ is not selected. As discussed above, the constructs in this consensus grid are arbitrary dimensions and are therefore uninformative. Once these options are set, ‘OK’ can be selected, and the ‘OK’ button of the Principal Components window can be selected to run the PCA. When the ‘OK’ button is pressed for the PCA, the user will be prompted to save the component score coefficients:



‘Yes’ should be selected, and then the user will provide a name for file to which the score coefficients will be saved. These score coefficients are essentially the numerical definitions of the principal components that result from the analysis. They will later be used to reconstruct the components when mapping the managers’ constructs. The following text output, without the annotation, will then be generated for the PCA by Idiogrid:

Principal Components Analyses (Correlations) for Consensus Grid

Eigenvalues for Unrotated Components

	Eigenvalue	% Variance	Cumulative %	Scree
PC_ 1	3.42	56.94	56.94	|**********
PC_ 2	1.24	20.63	77.57	|*****
PC_ 3	0.91	15.18	92.74	|****
PC_ 4	0.37	6.13	98.87	|**
PC_ 5	0.07	1.13	100.00	|*

Element Loadings (Unrotated)

	PC_1	PC_2
John	-0.97	-0.09
Bob	0.47	-0.25
Amy	-0.61	0.44
Jan	0.60	-0.73
Fred	1.09	0.66
Jill	-0.57	-0.03

Note. Values used for plotting elements in unrotated component space.

{Graph Created: Consensus Grid / PC_1 vs. PC_2 (Unrotated)}

The following graph will also be generated:

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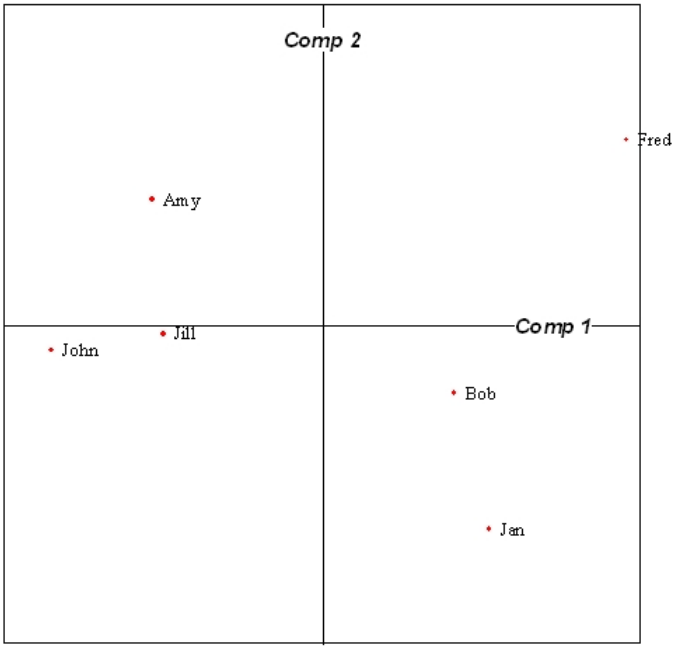
PCA (no rotation) for Consensus Grid

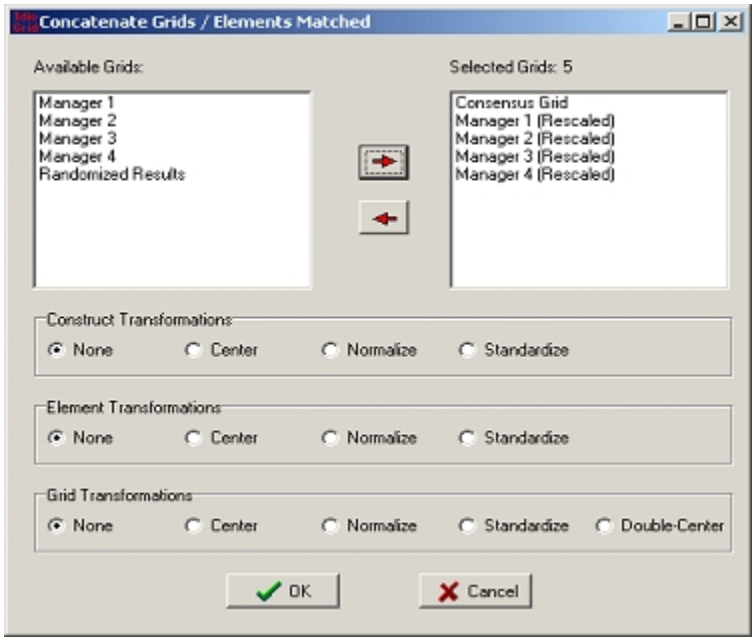
Axis Range: -1.14 to 1.14



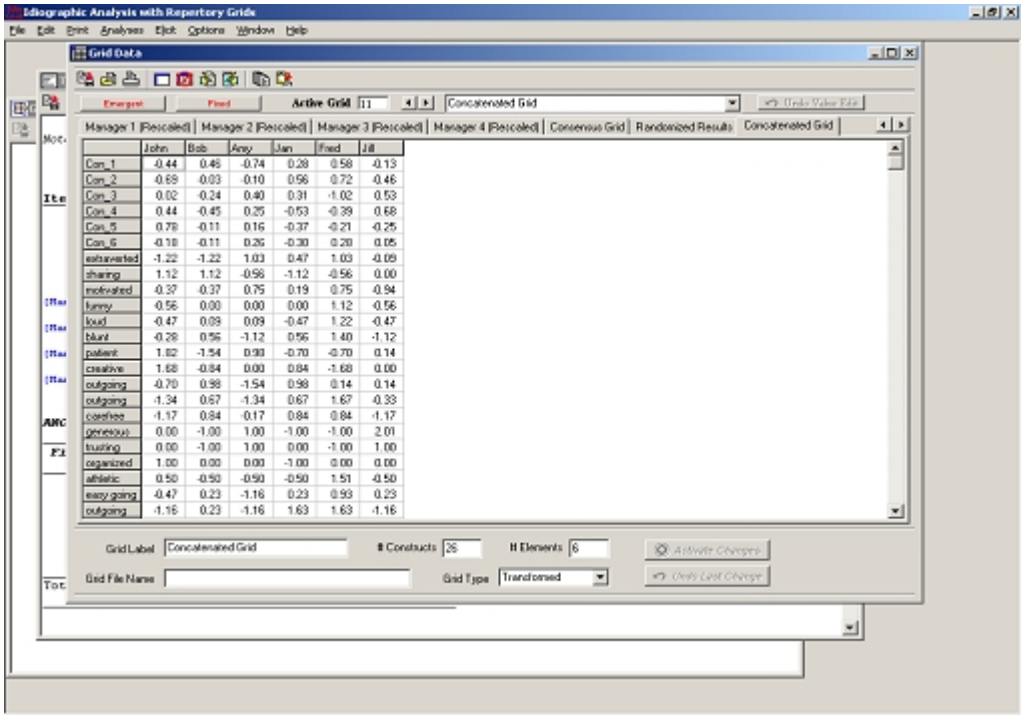
As can be seen in the graph of the first two components, which explain ~78% of the variance in the consensus grid (see the scree plot in the printed PCA output above), Amy, John, and Jill are similar to one another, and different from Bob and Jan, who are similar to one another. Fred is more similar to Bob and Jan than the other three employees, but he seems to be 'in a class by himself.'

Now the constructs will be added to the space using the Multiple Groups Components Analysis option in Idiogrid to conduct an extension analysis. The analyses below can be conducted for any single manager's grid or a subset of the manager's grids; however, all of the grids will be analyzed in this example.

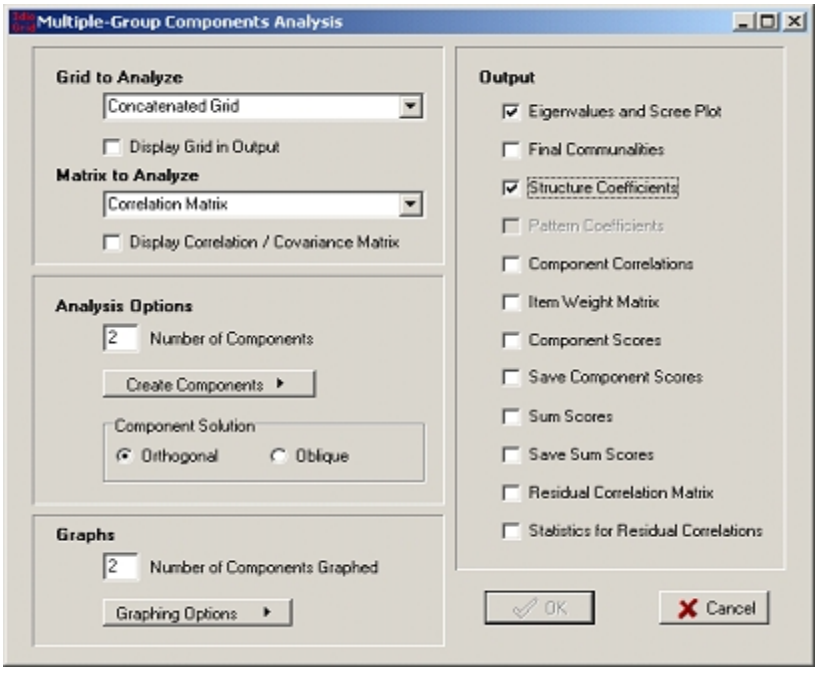
A concatenated grid is first created by making the **Grid Data** window visible in Idiogrid, and then selecting *Edit –> Concatenate Grids –> Elements Matched* from the Main Menu. The following window opens:



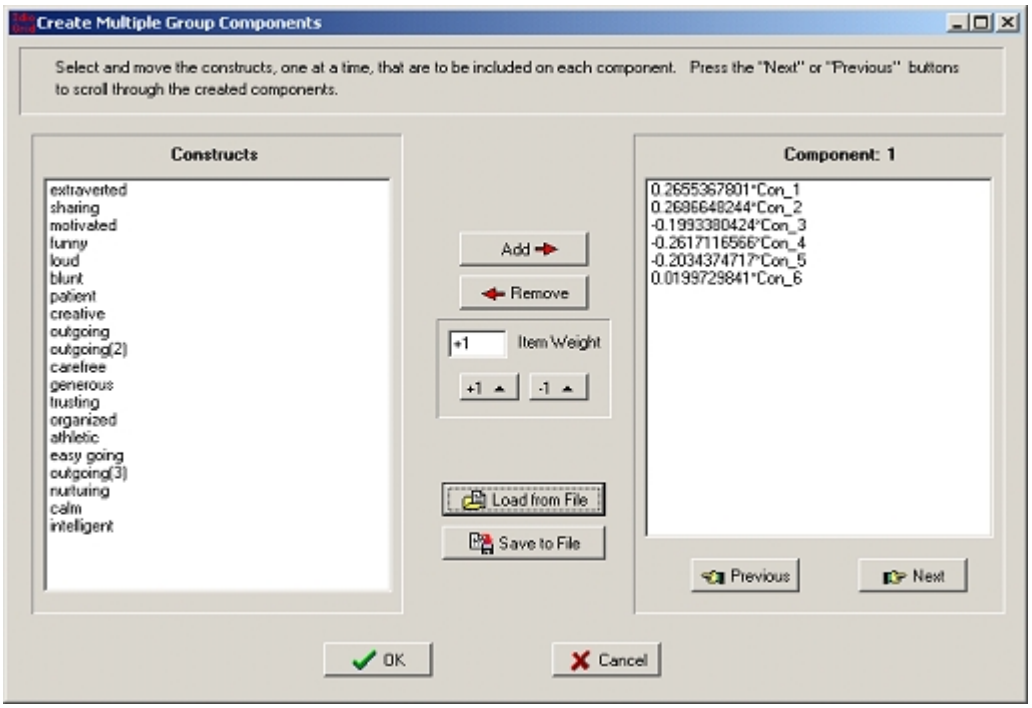
As can be seen, the consensus grid and the four managers' rescaled grids are moved into the right 'Selected Grids' window. **It is extremely important that the consensus grid be listed first when the grids are selected and moved into the 'Selected Grids' window.** If one of the manager's rescaled grids is listed first, the extension analysis below will be incorrect. Once the grids are selected, different transformations can be selected (none are selected here), and the 'OK' button pressed to generated the concatenated grid:



As can be seen in the concatenated grid above the first six rows are comprised of the consensus grid, and the remaining rows are the four managers' rescaled grids. Now the components that were created earlier in the PCA must be re-constructed and the managers' constructs mapped in the space. This is what methodologists refer to as an extension analysis. *Analysis –> Multiple Group Components Analysis* is selected from the Main Menu opening the following window:



Multiple Group Components Analysis is essentially a confirmatory type of analysis in which a researcher or practitioner can test the fit of a component model to the grid data. Here the analysis will be used to reconstruct the components extracted previously from the consensus grid so that the managers' constructs can be mapped into the space. In the options window above the concatenated grid is selected as the 'Grid to Analyze', the 'Number of Components Graphed' is set to '2', and the 'Eigenvalues and Scree Plot' and 'Structure Coefficients' options are selected. The 'Create Components' button is then selected to open the following window:



In the window above, the 'Load from File' button was pressed, and the component score coefficients that were saved from the PCA above were located and loaded. It can be seen that these coefficients are essentially multiplicative weights that are applied to the 'Con_1', 'Con_2', etc. dimensions to re-create the components. The 'OK' button is then pressed to return to the Multiple Groups Components options window. The 'OK' button is then selected to run the Multiple Groups analysis, which generates the following output:

Multiple-Group Components Analysis (correlations) for Concatenated Grid

Eigenvalues for Constructed Components

	Eigenvalue	% Variance	Cumulative %	Scree
C_ 1	13.87	53.33	53.33	|**********
C_ 2	4.44	17.08	70.41	|****

Structure Coefficients

	C_1	C_2
Con_1	0.91	-0.17
Con_2	0.92	0.08
Con_3	-0.68	-0.48
Con_4	-0.89	0.23
Con_5	-0.70	0.16
Con_6	0.07	0.95
extraverted	0.34	0.47
sharing	-0.39	-0.08
motivated	0.42	0.50
funny	0.82	0.53
loud	0.65	0.75
blunt	0.89	-0.02
patient	-0.85	0.23
creative	-0.68	-0.56
outgoing	0.68	-0.61
outgoing	0.97	0.00
carefree	0.91	-0.08
generous	-0.75	0.22
trusting	-0.74	0.00
organized	-0.60	0.41
athletic	0.35	0.59
easy going	0.77	-0.05
outgoing	0.95	-0.16
nurturing	-0.50	-0.48
calm	-0.91	0.29
intelligent	-0.75	0.47

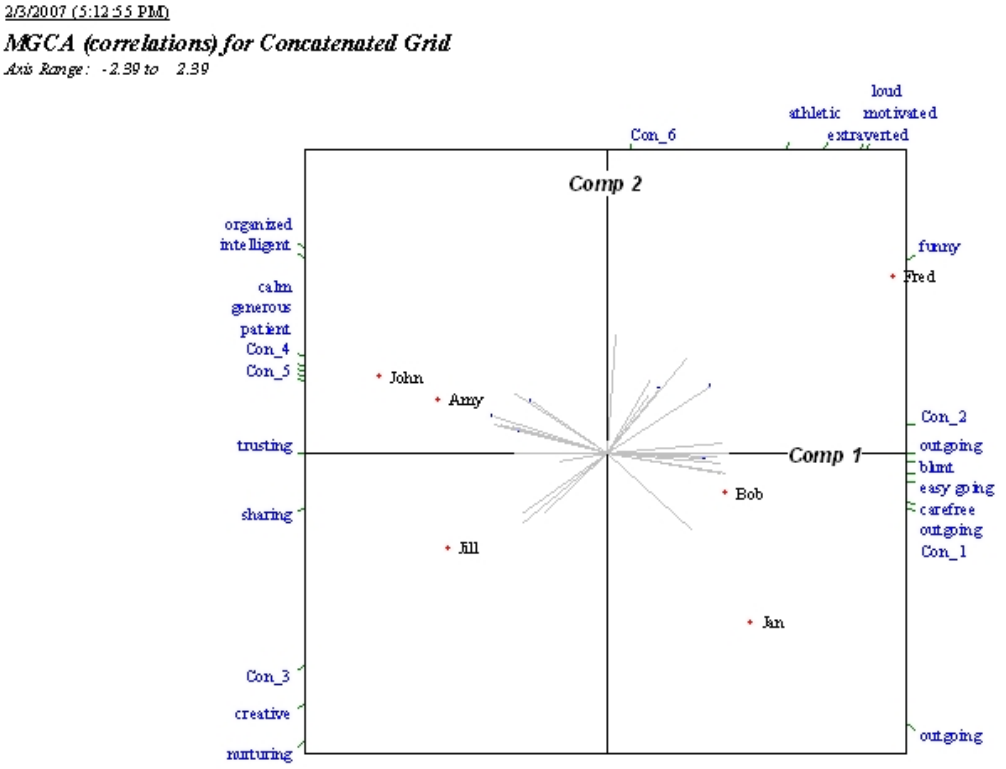
Element Loadings

	C_1	C_2
John	-1.79	0.60
Bob	0.94	-0.32
Amy	-1.32	0.42
Jan	1.15	-1.33
Fred	2.28	1.39
Jill	-1.25	-0.75

Note. Values used for plotting elements in component space.

{Graph Created: Concatenated Grid / C_1 vs. C_2 (MGCA)}

The following graph is also created:



It can be seen that Fred is ‘funny’, ‘extraverted’, ‘loud’, etc., whereas Jill is ‘creative’ and ‘nurturing.’ A common summary strategy in GPA is to examine the structure coefficients and tally constructs for each dimension. For example, the structure coefficients for this analyses are:

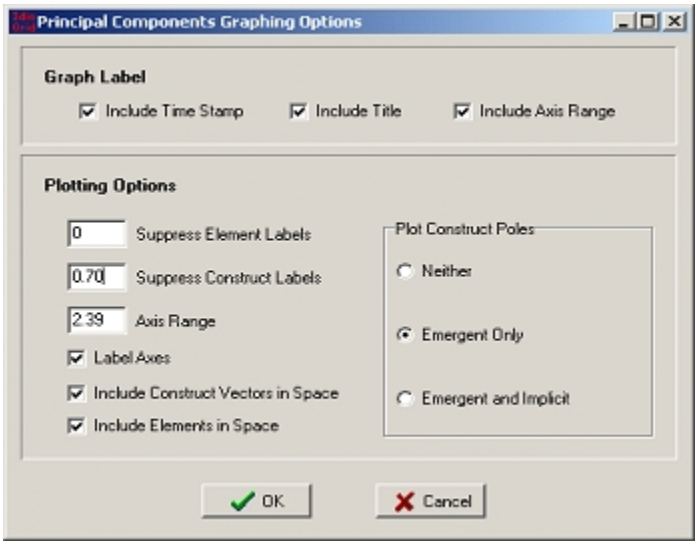
Structure Coefficients

	C_1	C_2
Con_1	0.91	-0.17
Con_2	0.92	0.08
Con_3	-0.68	-0.48
Con_4	-0.89	0.23
Con_5	-0.70	0.16
Con_6	0.07	0.95
extraverted	0.34	0.47
sharing	-0.39	-0.08
motivated	0.42	0.50
funny	**0.82**	0.53
loud	0.65	**0.75**
blunt	**0.89**	-0.02
patient	**-0.85**	0.23
creative	-0.68	-0.56
outgoing	0.68	-0.61
outgoing	**0.97**	0.00
carefree	**0.91**	-0.08

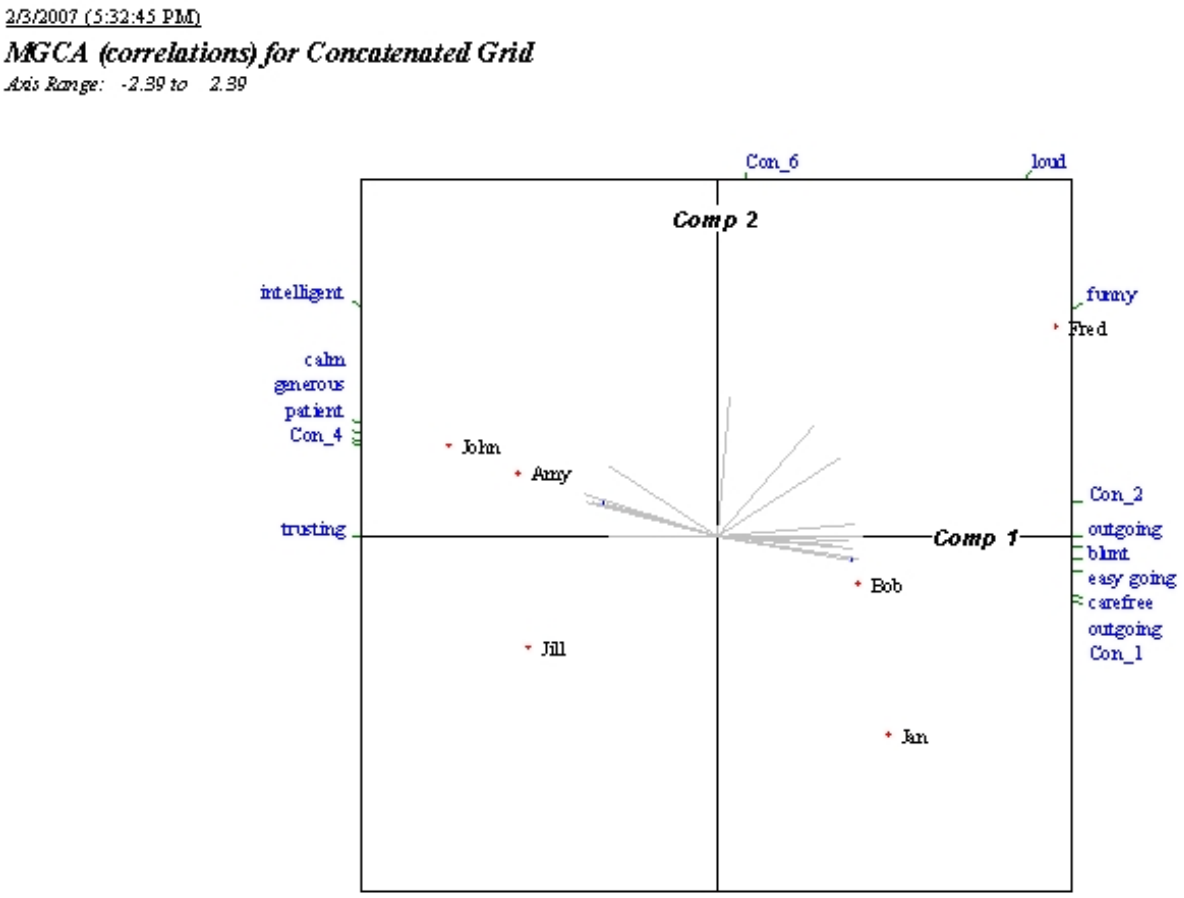
generous	**-0.75**	0.22
trusting	**-0.74**	0.00
organized	-0.60	0.41
athletic	0.35	0.59
easy going	**0.77**	-0.05
outgoing	**0.95**	-0.16
nurturing	-0.50	-0.48
calm	**-0.91**	0.29
intelligent	**-0.75**	0.47

A cut-point for salience must be determined; for instance, .70 (common values are 0.30, 0.35, and 0.40). Structure coefficients are correlations, and a cut-point of .70 would indicate at least 50% ($.70^2$) overlap between the constructs and the dimensions. The constructs that equal or exceed 0.70 in absolute value are printed in bold above. The first dimension therefore appears to be characterized primarily by being 'funny', 'blunt', 'outgoing', and 'carefree' compared to being 'patient', 'generous', 'trusting', 'calm', and 'intelligent. ' The second component only has one salient construct, 'loud.'

The non-salient constructs can be removed from the graph in Idiogrid. While viewing the graph in the **Graphics Output** window, select *Edit –> Change Options* from the Main Menu. The following options window will open:



As can be seen, the 'Suppress Construct Labels' option has been set to 0.70. Consequently, when the 'OK' button is selected a new graph will be created in which any constructs with structure coefficients less than 0.70 in absolute value will be omitted, as follows:



As can be seen in this new graph, only the salient constructs are shown. Finally, any graph in Idiogrid can be saved and then inserted into Powerpoint, Word, or most other Microsoft software. Once the graph is inserted in Powerpoint, for instance, the user will need to ungroup the objects in the graph, which can then be edited. In this way, the constructs, elements, vectors, etc. in the 2-dimensional plot can be moved, edited, deleted, etc. Editing the graphs in Microsoft software is particularly useful for deleting the arbitrary ‘Con_1', ‘Con_2', etc. dimension labels that show up as constructs in the extended plots above.

In published papers of GPA results, one can often find the output and graphs like those above. With only a little editing outside of Idiogrid, the results and graphs should be ready for presentation or publication.